Mark Meerschaert (1955-2020)

Paul Anderson, Alla Sikorskii, Erkan Nane, Farzad Sabzikar

Figure 1: Mark Meerschaert, seated, at the *Workshop on Future Directions in Fractional Calculus Research and Applications*, which Mark organized at Michigan State University

Mark Meerschaert passed away on August 29, 2020 shortly after his 65th birthday, following a courageous battle with cancer. Mark made seminal contributions to multiple areas of applied probability with applications to several areas of science, and he had a profound influence on colleagues whom he collaborated with and mentored. His kind and optimistic nature, his devotion to mathematics and its applications, including nurturing the next generation of mathematicians, and his devotion to his family were clearly apparent to those who knew Mark. Remembrances below from four of Mark's colleagues give more detail on his scholarly work and his efforts and gifts as a mentor and colleague.

Mark studied mathematics at the University of Michigan, earning a BS in 1977, a MS in 1979, and a PhD in 1984. Mark's doctoral research, resulting in a dissertation titled *Multivariable Domains of Attraction and Regular Variation*, was conducted under Fred Hoppe. Mark's academic work took him to several diverse institutions, including Albion College in Albion, Michigan (USA), The University of Nevada in Reno, Nevada (USA), The University of Otago

in Dunedin (NZ), and most recently Michigan State University in East Lansing, Michigan (USA). Mark's appointments at these institutions give a sense of his intellectual breadth, with appointments as Professor of Mathematics, Professor of Physics, Faculty in Hydrologic Sciences, and Professor of Statistics and Probability.

Mark's scholarly contributions were recognized by the title of University Distinguished Professor at Michigan State University, a prestigious and rare title awarded to faculty who are internationally recognized for the importance of their achievements. Mark also provided leadership while continuing his impressive research and mentoring work as Chair of Applied Mathematics at The University of Otago and as Chair of Statistics and Probability at Michigan State University.

Mark's formal connection with the Department of Statistics and Probability at Michigan State University goes back at least to 1992, when Mark visited the department while on sabbatical from Albion College. Mark returned to the department in 2006 as Chairperson, and stayed in the department until his retirement in 2020.

Mark was diagnosed with prostate cancer in 2013. He brought the same focus to that challenge as he did to his research. A brief illustrative story can be found at Treating the Gene. Over the past several years, it became very clear to anyone who interacted with Mark that his commitment to his scholarly work, his kindness and optimism, and his devotion to his family were absolutely central to Mark and were not shaken by his diagnosis and subsequent treatment. Mark continued to produce exceptional scholarly work, always had a smile and a kind word for those he interacted with, and of course saved his biggest smiles for when he talked about his family and especially his grandchildren!

Remembering Mark Meerschaert

Paul Anderson

Mark was largely responsible for my hiring at Albion College in 1990. Fortunately for me, beginning in 1993, we combined my background in time series analysis with his background in probability theory to write papers that characterized periodically stationary time series. Our first paper landed in *The Annals of Statistics*. The main result of that paper showed that when the time series had finite variance but infinite fourth moment, the sample autocorrelations are asymptotically stable. It is well known in this case that sample autocorrelations for classical stationary moving averages are asymptotically normal. Prior to writing this paper, I had no knowledge of what a stable limit was, of what regularly varying tails with index α meant, of what a domain of attraction was, etc., etc. Mark patiently spent hours on end with me, presenting lectures on probability theoretic results pertinent to the work in writing and completing our paper. His ability to take an advanced topic and explain it to a novice so that it had real meaning was astounding. It was easy to see that not only was Mark a gifted researcher, he was even a more gifted teacher. His energy for doing mathematics was infectious and we had so much fun doing research together, especially on our first paper together.

There is one research moment that has always stuck with me, as in, I remember it like it was yesterday. It occurred during 1998 late winter at the University of Nevada in Reno. Mark and I needed to bound the matrix-2 norm of autocovariance matrices of increasing dimension using bounds from a spectral density matrix of a vector ARMA process, $Y_t = (X_{t\nu}, X_{t\nu+1}, \ldots, X_{t\nu+\nu-1})'$ where X_t is periodi-

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cally stationary with period ν. Being a probability theorist, Mark had never seen a spectral density matrix of a multivariate time series. That was at 9 am that morning. He looked at Definition 11.6.1 in Brockwell and Davis', *Time Series: Theory and Methods* (1991), and then we found an empty classroom with many blackboards. Mark's only ammunition consisted of Definition 11.6.1 and a piece of chalk. Without any notes he meticulously navigated a path to our desired result through the relationship of the spectral density function and the autocovariance function. He created a super-covariance matrix Γ where if we let $\Gamma(h) = Cov(Y_t, Y_{t+h})$, $Y = (Y_{n-1}, \ldots, Y_0)'$, then $\Gamma = Cov(Y, Y) = [\Gamma(i-j)]_{i,j=0}^{n-1}$, where n is as large an integer as you like. Mark explained every step along the way as I was taking notes and nodding my head in approval. At the end of this session, about 1 pm, we broke for lunch. I asked Mark how he was able to do all of that from scratch and no notes, and he said he had been working in multiple dimensions his whole life and this was another application of that. For four hours I had been mesmerized by mathematical virtuosity. For me, there will always be a modicum of disbelief at what I witnessed that day. No notes, just chalk, and pure genius.

Certainly, I look back on all of the projects that Mark and I worked on and completed together with fondness and pride. I could not have written these papers without Mark, but he always gave me the credit when the job was finished. Mark was a mentor and friend, and it is his friendship that I most cherish. From the beginning, he treated me like a brother. Indeed, I felt like one of the family. In Reno, Nevada when the Meerschaerts went out to dinner, guess who was along with them. Every now and then, when we took on Mark's sons, Bob and Steve, in two-on-two basketball, they would let Mark and me win. Mark's beautiful wife, Carmen, had me

over to dinner about a thousand times and all I was asked to do was have seconds and another beer–that's living folks!

Figure 2: Paul Anderson, Mark Meerschaert, and a redwood tree

During his extraordinary career in teaching and researching mathematics, Mark touched many lives similarly to the way he touched mine. He made everyone feel special because he thought everyone was special. We worked on one last paper together this past summer with our colleague, Farzad Sabzikar. I asked Mark if he felt up to working on this paper, a paper on modelling climate data, and he assured me that he looked forward to it. The hours we spent in April and May ironing out the details boggles my mind when I think about it. He gave the paper a luster it would never have had if not for his brilliant input. Thank you, Mark, for in-

dulging me one last time on another epic project. I am happy that we got to visit some together and go get coffee and a pastry. I will surely miss all of these times. God bless you, Mark M. Meerschaert.

Remembering Mark

Alla Sikorskii

I do not have the right words to describe the loss of Mark. In what follows, I did my best to mention Mark's accomplishments and people connected to the Department of Statistics and Probability (Department) who had benefited from knowing him. His death was a loss for the Department, but more importantly, it was loss for many people whose lives had been touched by Mark in ways so significant that the words fade. "FADE" happens to be the acronym for the fractional advection-dispersion equation, researched by Mark and his co-authors. The first time I saw the term was in Paramita Chakraborty's presentation of her thesis, directed by Atma Mandrekar, during the colloquium at the Department more than 10 years ago. The presentation was superb, and Paramita, Mark, and Chae Young Lim published the results on parameter estimation of fractional transport in *Water Resources Research* in 2009. This is just one reflection of Mark's influence that shaped the scientific advancement of many students, whether or not these students were formally directed by him as a major professor. One of Mark's former students, Farzad Sabzikar and I have written a memorial note for the *Fractional Calculus and Analysis* journal, 'Ed. Note, FCAA–Volume 23–5–2020': `https://www.degruyter.com/view/journals/fca/23/5/article-p1241.xml` Within the page

limits, Farzad and I only touched on the fundamental contributions that Mark made to the field of fractional calculus and its connections to partial differential equations, probability, and statistics.

These connections inspired me when I listened to Mark's lectures in the graduate course in Fall of 2010. One lecture was enough for me to decide that I wanted to work in the field of fractional calculus. Our book, M. M. Meerschaert and A. Sikorskii, *Stochastic Models for Fractional Calculus*, De Gruyter Studies in Mathematics Vol. 43, Second Edition, 2019, ISBN 978-3-11-056024-4, `https://www.degruyter.com/view/title/533933` stemmed from this course, with the goal of introducing people to the beautiful world of fractional calculus. Mark insisted that the book had to be written in a way of a rigorous presentation that people can read easily. The format of the book of the introduction of main ideas followed by a technical subsection with details has been very successful. Others have followed this format and almost verbatim copied other text we wrote, and Mark's comment was: "They say that imitation is the sincerest form of flattery. This is the preface of a new book `https://www.degruyter.com/view/title/539917` and if you compare to the preface in our book, you will see what I mean." This comment was so characteristic of Mark who always managed to find positive things and focus on them. Deep into his battle with cancer, Mark enjoyed working on the second edition of the book. I hope it will be useful to people who want to enter and continue their work in fractional calculus. Farzad Sabzikar read and re-read the drafts of the book, with our sincere thanks! He also used the results from the book in his work when he took a new and very interesting direction of introducing and investigating tempered fractional Brownian motion. Mark was very

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proud of Farzad, especially of how Farzad created a new research topic. Mark's other PhD student from the Department, Kai Zhang, worked in an area of modeling and forecasting using time series. When Mark's grandson was born, Mark was very excited: "My grandson's name is Kai, same as my PhD student's!".

Being readable by and useful to students is also a key feature of one of Mark's other books, the textbook for undergraduate students that had 4 editions: M. M. Meerschaert, *Mathematical Modeling*, 4th Edition (2013), Academic Press, ISBN 978-0-12-386912-8. Mark's ability to learn applications and make a sustained impact on research in applications of fractional calculus, statistics, and probability, elevated many researchers from other fields such as hydrology or geology. In 2016, he organized a workshop on future directions in fractional calculus research and applications. The workshop was held at the Department:

    https://stt.msu.edu/FCworkshop/

and was attended by people from many disciplines. It was such a pleasure listening to people's presentations and real stories about how mathematical models of real-world phenomena can be not only useful, but crucial for the decisions made to protect water supplies and people. James Kelly, MSU post-doc mentored by Mark at the time, won the prize for the best presentation on the cutting-edge work on non-local but not necessarily fractional problems.

On the subject of mentoring, I have a memory of Mark that makes me smile every time I think of it. Around 2010, MSU departments were told to come up with formal mentoring arrangements for junior faculty, mentoring plans, and schedules of mentoring meetings. Mark's said that if one wanted to effectively mentor a junior colleague, they should write a paper with them. Meetings and discussions would then come naturally. This is exactly what happened

when I wrote papers with Mark. I learned a lot, and now I apply this mentoring approach to my mentees. We write papers and grants, then implement grants together, then write more papers.

Figure 3: Mark Meerschaert at the International Symposium on Fractional PDEs: Theory, Numerics and Applications

Mark's third book, co-authored with Hans-Peter Scheffler, M. M. Meerschaert, H. P. Scheffler, *Limit Distributions for Sums of Independent Random Vectors: Heavy Tails in Theory and Practice*, Wiley Ser. in Probability and Mathematical Statistics, 2001, ISBN 0-471-35629-8, laid the foundation for many papers that connected statistics, probability, partial differential equations, and fractional calculus. To mention just some, in M. M. Meerschaert and H. P. Scheffler, Limit theorems for continuous time random walks with infinite mean waiting times, *Journal of Applied Probability* **41**, No 3 (2004), 623–638, the authors extended the limit theory to continuous time random walks (CTRWs), where independent identically distributed (iid) particle jumps are separated by iid waiting times. When waiting times have a power law distribution with exponent between 0 and 1, the CRTW has a

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weak limit of Brownian motion time-changed to the inverse stable subordinator.

The connection of the inverse stable subordinator to fractional calculus is another important example of cross-disciplinary research conducted by Mark. He has an enormous number of citations for his papers, but the one cited the most in my papers with Mark was his paper with Boris Baeumer: B. Baeumer and M. M. Meerschaert, Stochastic solutions for fractional Cauchy problems, *Fract. Calc. Appl. Anal.* **4**, No 4 (2001), 4811–500. Theorem 3.1 of this paper is a purely analytical result connecting solutions of a fractional and corresponding non-fractional Cauchy problems. Mark mentioned that at that time, he was not thinking about the connection to the inverse stable subordinator. The connection became clear later, with a fruitful line of research on stochastic solutions of fractional partial differential equations that can be obtained via a time change of the solution of the non-fractional equation to the inverse stable subordinator.
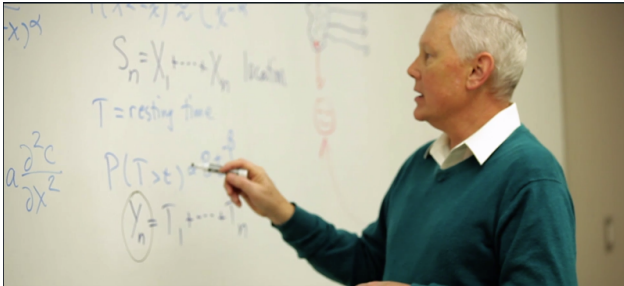

Figure 4: Mark giving a lecture

Mark's research legacy of over 200 publications is amazingly broad and includes papers in the *Annals of Statistics* (with Paul Anderson who at one time was senior teaching specialist in the Department), *Annals of Probability* (five papers, one with Peter Straka who was a post-doc in the Department, another with Erkan Nane and P. Vellaisamy who were visiting faculty in the Department), *Transactions of the American Mathematical Society*, *Journal of Mathematical Analysis and Applications*, to name a few, and journals in fields other than Statistics, Probability, or Mathematics. His most cited paper M. M. Meerschaert and C. Tadjeran, Finite difference approximations for fractional advection-dispersion flow equations, *Journal of Computational and Applied Mathematics* **172**, No 1 (2004), 65–77, is on numerical methods for FADE. This paper is an example of how it takes both science and art to develop useful numerical methods. While the approximation of fractional derivative using standard Grünwald's formula should work based on theoretical considerations, it does not in practical approximation schemes for fractional partial differential equations. The art part was then to think of a shifted Grünwald's approximation, and prove scientifically that it works, which is what was done in that paper.

Mark's and my collaborator Nikolai Leonenko once used an analogy with an ancient myth about Earth resting on the backs of whales. Nikolai wrote that Kyiv school of probability and mathematical statistics had rested on the backs of two whales, the genius of Anatoli Skorokhod and warmth of the soul of Mikhailo Yadrenko. Since I joined the Department as a faculty in 2008, in my perception, the Department rested on the back of one great whale: Mark Meerschaert, who combined both the outstanding mind of a brilliant scientist and warmth of a person who cared about others and made every day bright and sunny for those around him.

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A Great Mentor and Friend

Erkan Nane

I met Mark first time in August 2006. He accepted the position of Chair of Department of Statistics and Probability, and I was hired as a visiting assistant professor. My research direction almost totally changed after I started my position at MSU and met Mark.

I was invited to give a colloquium talk at the Department of Statistics and Probability early September 2006. At this talk, Mark asked several questions about my research on iterated Brownian motions. Later during the Fall semester of 2006, we had discussed the problems I am interested in and the research area he was interested in during lunch time in the Department Lounge. By training I was coming from a background in the probabilistic approach to Analysis, and his research area had connections to down to earth topics modeling natural phenomena with time fractional equations and time changed processes. So, our first paper was connecting in a sense the two areas of approach-iterated processes and time fractional equations. Mark was very patient with me as I have not studied any real applications and did not know well how to explain the methods. But Mark asked me very critical questions and guided me towards an understanding of his research area. Our first paper led to a fruitful collaboration with Mark in the two-years I stayed at MSU and all my career after I left MSU.

Mark was very kind to me. He was very enthusiastic about math research. He was open and eager to work on new areas of research topics. He did not insist to work on research only in his area. On one occasion, we discussed a topic with a visitor P. Vellaisamy and proposed to work on this topic with Mark. He got excited and we set up weekly meetings to fill in the details of the first paper. This paper was finalized during my busiest time of job hunting in early Spring semester of 2008. From our simple idea of the paper, with Mark's direction and guidance to add a detail here and an approach there, the paper was accepted at *Annals of Probability*. Later we got 4 more papers together with P Vellaisamy and Mark.

The year I was on the job market, despite the fact that Mark already extended my contract for 3 more years at MSU, he encouraged me to apply for jobs. He helped me in my application materials to the minute details and gave me critical advice to go to a job interview.

Figure 5: Mark Meerschaert

I worked with Mark actively up until 2013. During the NSF/CBMS Conference on "Analysis of Stochastic Partial Differential Equations" Michigan State University (August 19–23, 2013), I had a chance to have lunch and chat

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with him. He advised me to work on an area of research to further advance my career. I was not happy with this as I wanted to stay close to him. This actually helped me realize my potential and I have done a large amount of good research with his direction.

Even after I left MSU in 2008 and started my job at Auburn University, he would be kind to help me with my NSF grant applications, tenure, and promotion application. He was encouraging all the time and gave fair and to the point advice to advance my career in Academia.

My first son Asim (second child), was born in Lansing in 2006 during my stay at MSU. In one occasion at the lounge, Mark hold my son and gave his whole attention to my son and Mark found a way to entertain him with a plastic spoon: my son was happily chewing/biting on a plastic spoon.

I cannot finish without our visit to Mark's house, and the barbecue fish and other delicious food Carmen prepared for us. I knew Mark worked very hard, and felt close enough to ask him about how he can manage work and family. He advised me to give full attention to wife or children whenever they ask for. Since then, this advice I see priceless, helped me with my work-family balance.

Last time I saw Mark was at the Workshop on Future Directions in Fractional Calculus Research and Applications. MSU, October 17-21, 2016. He was one of the hosts of this workshop and was having hard time to stand up for long because of the treatment. He still was lively and enthusiastic about research on fractional calculus. After this meeting I have asked some advice from Mark about grant applications and promotion via e-mail. He gave me his best advice despite fighting cancer.

All in all, Mark made a lasting impact in fractional calculus and in the use fractional calculus for modelling natural phenomena in many areas including Hydrology, Physics, Engineering, Geology to name a few of the many areas.

I cannot appreciate his influence enough on my career as well as on my life. I think people who worked with him can attest to his influence with all their hearts.

It was an honor to have known Mark. I will sorely miss him.

Remembering a Great Mentor
Farzad Sabzikar

In the following note I describe Mark M. Meerschaert as a PhD advisor to me, his graduate student. I think the note would be useful for junior faculties who are in earlier steps of their career.

I met Mark for the first time in July 2009 when I was seated in a class for new grad students at Michigan State University (MSU). He came to the class around noon with a pleasant and kind face. Then welcomed all the students and introduced himself. Unlike most mathematicians, he sounded very friendly and familiar at the first glance. His behavior gave me the impression that I have known him for many years. He discussed many topics that day, and I remember the most important ones: being respectful, being on time, and making eye contact with all the students. That was actually the first lesson I learned from him.

I met Mark again in spring 2010 when he came to my office and offered for having cappuccino together. After a few days we walked through the beautiful campus to Mark's favorite coffee shop. Mark asked me about my research interest first, then he started explaining his research area which was mainly related to the frac-

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tional calculus. Our discussion was fascinating to me since I could see the connection between fractional calculus and my knowledge of stochastic processes. On the same day, Mark told me that he and Professor Alla Sikorski were planning to write a book on fractional calculus and offered me to read the book. His goal was to employ a graduate student for reading the book so he would have an idea of how potentially useful the book is for students and researchers outside the probability and statistics area.

Figure 6: Mark Meerschaert and Farzad Sabzikar after Farzad's doctoral defense

My first attempt for the Ph.D. qualification exam in the following summer was not successful so I had to take my second and last chance in January 2011. Meanwhile, Mark and I had our third meeting in his office. He shared with me his story of being unsuccessful on his first Ph.D. exam and then how he succeeded the sec-

ond time with achieving the highest score among all other students. He was proud that he prepared for this exam while he helped his spouse in taking care of their young children. Hearing his story was a powerful motivation for me to move forward and overcome the exam. This was another lesson I learned from Mark, encouraging the students to continue their hard work and helping them to stay motivated after they receive an unsuccessful result.

My fourth meeting with Mark was on January 2011 when I officially requested him to serve as my PhD mentor. Now that was time for me to read the book that Mark had offered and familiarize myself with his point of view in fractional calculus. We hold weekly meetings to discuss each section of the book and during one of those meetings, Mark taught me the idea of tempered fractional calculus which became the backbone of my research career until now. Although most of the time of our meetings were dedicated to the research, he sometimes used to talk to me about real life which is beyond research. He advised me to while staying focused and remaining productive in my career, I should manage to spend time with my family and enjoy other aspects of life. In fact, taking this advice is important especially for international students who are experiencing a new culture and environment. I personally took his advice and after I became a mentor I pass it to my students as well.

Later on in February 2012, Mark suggested me to attend in the international conference held in honor of Professor Murad S. Taqqu and participate in poster presentation session. When we met to discuss about my poster and prepare for my presentation, he provided me some math comments first, then he gave me another take home message: Be respectful to all participants in the conference and make sure you come up with correct answers when they question about your poster.

I got married in the Summer of 2012 in Iran. When my wife and I came back to the U.S. Mark invited us for lunch. He even cared about our culture in that he chose a Mediterranean restaurant which their menu was similar to our traditional meals and he did that mostly because of my wife who was new to the U.S. Welcoming her this way was another great lesson I learned from Mark. The main point to always remember is: You want to have a happy wife and dedicate the weekends and holidays to your family.

Lastly, my Ph.D. defense was on May 5, 2014, and I received my degree under Mark's supervision. After the defense I went to his office to thank all his supports during my graduate studies. I told him that I am still his student and will be always learning from him, and he responded "No. You are not my student anymore. You are my colleague and collaborator from now on". Mark was not my PhD mentor only, but the great support to me after I joined the Department of Statistics at Iowa State University in August 2015. He indeed was an excellent mentor in my life. We kept communicating and collaborating during all years after my graduation. Even though he was involved in his disease and struggled with its progression, he communicated with his collaborators including me until his last days. Although Mark is no longer with us physically, he will always stay with us in our heart and mind. The last written statement I learned from Mark is the following: "Farzad, If I have done you any good, please pay it forward to the next generation of students."