2023 Workshop on Stochastic Analysis, Random Fields, and Applications

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Tutorial Lectures

Davar Khoshnevisan (University of Utah, U.S.A.)

Tutorial I. An Introduction to Parabolic SPDEs

In the first expository tutorial we outline portions of the basic theory of stochastic partial differential equations for parabolic equations of Itô type. Our discussion includes mainly an introduction to well-posedness and regularity theory for classical SPDEs.

Tutorial II. Blowup in Parabolic SPDEs

In the second tutorial we summarize a few of the recent advances in the study of finite-time blowup for nonlinear SPDEs of parabolic type.

Invited Talks (In alphabetic order of the first names)

The Boltzmann Processes

Barbara Rüdiger (Bergische Universität Wuppertal, Germany)

Abstract: Given a solution $f := \{f(t, x, v)\}$ of the Boltzmann equation we prove the existence of an associated stochastic differential equation driven by a Poisson noise that depends on f. We show that, when the distribution of the solution admits a probability density for each time t, then this coincides with f. The stochastic differential equation is hence of McKean -Vlasov type and its solution is a "Boltzman process", i.e a stochastic process which describes the dynamic in space and velocity of a tagged particle of a gas evolving with density f. We construct in this way Boltzmann processes for the Boltzmann equation with hard spheres and with hard potentials. The proof relies on first showing that any solution f of the Boltzmann equation is also the unique solution of an associated bilinear equation. This is a joint work with S. Albeverio and P. Sundar.

The Radius of Star Polymers in Low Dimensions and for Small Time

Carl Mueller (University of Rochester, U.S.A.)

Abstract: Studying the end-to-end distance of a self-avoiding or weakly self-avoiding random walk in two dimensions is a well known hard problem in probability and statistical physics. The conjecture is that the average end-to-end distance up to time n should be about $n^{3/4}$.

It would seem that studying more complicated models would be even harder, but we are able to make progress in one such model. A star polymer is a collection of N weakly mutually-avoiding Brownian

motions taking values in $\mathbf{R}^d$ and starting at the origin. We study the two and three dimensional cases, and our sharpest results are for $d = 2$. Instead of the end-to-end distance, we define a radius R_T which measures the spread of the entire configuration up to time T. There are two phases: a crowded phase for small values for T, and a sparser phase for large T where paths do not interfere much. Our main result states for $T < N$, that R_T is approximately proportional to $T^{3/4}$ and also to $N^{1/4}$.

Lifting of Volterra Processes: Optimal Control in UMD Banach Spaces
Giulia Di Nunno (University of Oslo, Norway)

Abstract: We study a stochastic control problem for a Volterra-type controlled forward equation with past dependence obtained via convolution with a deterministic kernel. To be able to apply dynamic programming to solve the problem, we lift it to infinite dimensions and we formulate a UMD Banach-valued Markovian problem, which is shown to be equivalent to the original finite-dimensional non-Markovian one. We characterize the optimal control for the infinite dimensional problem and show that this also characterizes the optimal control for the finite dimensional problem. The presentation is based on joint work with Michele Giordano (University of Oslo).

Gaussian Processes in the Era of Big Data: Strategies for Approximate Inferences
Hao Zhang (Purdue University, U.S.A.)

Abstract: Gaussian processes serve as a robust modeling framework, finding extensive use across various disciplines within statistical and machine learning applications. The kernel, or covariance matrix, is a pivotal component in modeling Gaussian processes, aiding in estimation and prediction. However, as the sample size expands, the kernel matrix tends to become ill-conditioned, necessitating approximation strategies for Gaussian likelihood or spatial prediction. In this talk, I will survey an array of such approximation methods and share recent findings in the field.

Nonlinear Random Perturbations of PDEs and Quasi-Linear Equations in Hilbert Spaces
Sandra Cerrai (University of Maryland, U.S.A.)

Abstract: We study a class of quasi-linear parabolic equations defined on a separable Hilbert space, depending on a small parameter in front of the second-order term. Through the nonlinear semigroup associated with such an equation, we introduce the corresponding SPDE and we study the asymptotic behavior of its solutions, depending on the small parameter. We show that a large deviations principle holds and we give an explicit description of the action functional.

Short Talks

Brownian Motion with Power Law Drift
Adina Oprisan (New Mexico State University)

Abstract: In this talk we will discuss the power law drift influence on the exit time of Brownian motion from the half-line. We show that the behavior of the process far away from zero has the greatest influence and we evaluate the lifetime of the process using large deviations techniques. Motivated by these results, we consider a modification of the power law drift with a slowly varying function. This talk is based on a joint work with D. DeBlassie and R. Smits.

Colored Noise and Parabolic Anderson Model on Torus

Cheng Ouyang (University of Illinois at Chicago)

Abstract: We construct an intrinsic family of Gaussian noises on compact Riemannian manifolds which we call the colored noise on manifolds. It consists of noises with a wide range of singularities. Using this family of noises, we study the parabolic Anderson model on compact manifolds. To begin with, we started our investigation on a flat torus and established existence and uniqueness of the solution, as well as some sharp bounds on the second moment of the solution.

Local Times of Gaussian Random Fields and Stochastic Heat Equation

Cheuk Yin Lee (Chinese University of Hong Kong, Shenzhen)

Abstract: In this talk, I will discuss the local times of anisotropic Gaussian random fields satisfying strong local nondeterminism with respect to an anisotropic metric. We prove local and global Holder conditions for the local times for these Gaussian random fields and deduce sample path properties related to Chungs law of the iterated logarithm and modulus of non-differentiability of the Gaussian random fields. Our results can be applied to systems of linear stochastic heat equations with additive Gaussian noise. We also determine the exact Hausdorff measure function for the level sets of the solution. This talk is based on joint work with Davar Khoshnevisan and Yimin Xiao.

Bifractional Brownian Motions on Metric Spaces

Chungsheng Ma (Wichita State University)

Abstract: Fractional and bifractional Brownian motions can be defined on a metric space, if the associated metric or distance function is conditionally negative definite (or of negative type). This talk introduces several forms of scalar or vector bifractional Brownian motions on various metric spaces, and presents their properties. A metric space of particular interest is the arccos-quasi-quadratic metric space over a subset of $\mathbb{R}^{d+1}$ such as an ellipsoidal surface, an ellipsoid, or a simplex, whose metric is the composition of arccosine and quasi-quadratic functions. Such a metric is not only conditionally negative definite but also a measure definite kernel, and the metric space incorporates several important cases in a unified framework so that it enables us to study (bi, tri, quadri)fractional Brownian motions on different metric spaces in a unified manner. The vector factional Brownian motion on the arccos-quasi-quadratic metric space enjoys an infinite series expansion in terms of spherical harmonics, and its covariance matrix function admits a ultraspherical polynomial expansion. We establish the property of strong local nondeterminism of fractional and bi(tri, quadri)fractional Brownian motions on the arccos-quasi-quadratic metric space.

Peak Height Distributions of Gaussian Random Fields and Their Applications in Statistics

Dan Cheng (Arizona State University)

Abstract: Motivated by computing p-values for multiple testing of local maxima in signal and change point detections, we study the height distribution of local maxima of smooth isotropic Gaussian random fields parameterized on Euclidean space or spheres. The obtained formulae hold in general in the sense that there are no restrictions on the covariance function of the field except for smoothness and isotropy. The results are based on a characterization of the distribution of the Hessian of the Gaussian field by means of the family of Gaussian orthogonally invariant (GOI) matrices, of which the Gaussian orthogonal ensemble (GOE) is a special case.

Continuity Properties of a Family of Time Fractional Stochastic Heat Equations

Erkan Nane (Auburn University)

Abstract: In this talk we present temporal continuity and continuity with respect to fractional order of the solution to a certain class of space-time fractional stochastic equations. Our results extend the main results in "M. Foondun, Remarks on a fractional-time stochastic equation, Proc. Amer. Math. Soc. 149 (2021), 2235-2247" and the main results in "D.D. Trong, E. Nane, N.D. Minh, N.H. Tuan. *Continuity of solutions of a class of fractional equations*
Potential Anal. 49 (2018), no. 3, 423–478".

These results are our recent joint work with Alemayehu G. Negash, Jebessa Mijena and Nguyen Huy Tuan.

Local Limit Theorem for Linear Random Fields

Hailin Sang (University of Mississippi)

Abstract: We establish local limit theorems for linear random fields when the i.i.d. innovations have finite second moment or the innovations have infinite second moment and belong to the domain of attraction of a stable law with index $0 < \alpha \leq 2$ under the condition that the innovations are centered if $1 < \alpha \leq 2$ and are symmetric if $\alpha = 1$. When the coefficients are absolutely summable we do not have restriction on the regions of summation. However, when the coefficients are not absolutely summable we add the variables on unions of rectangles and we impose regularity conditions on the coefficients depending on the number of rectangles considered. Our results are new also for the dimension 1, i.e. for linear sequences of random variables. The examples include the fractionally integrated processes for which the results of a simulation study is also included.

This talk is based on two papers jointly with Timothy Fortune, Magda Peligrad, Yimin Xiao and Guangyu Yang.

Does SLOPE Outperform LASSO?

Haolei Weng (Michigan State University)

Abstract: Existing results have showed that two popular estimators, SLOPE and LASSO, both achieve the minimax estimation rate under high-dimensional sparse linear regressions. However, such order-wise accurate results are insufficient to capture the performance difference of the two estimators. In this talk, we will showcase how Gaussian comparison theorems can be leveraged to derive sharp constant-wise accurate finite-sample error analysis. This refined analysis provides new insights into the performance of SLOPE and LASSO.

Multiplicative Stochastic Heat Equation on Compact Riemannian Manifolds of Nonpositive Curvature

Hongyi Chen (University of Illinois at Chicago)

Abstract: We establish well-posedness of and moment exponential upper bounds on the solution of the multiplicative stochastic heat equation on compact manifolds with measure-valued initial condition. Unlike most previous work on this equation, Fourier analysis is not employed. Existence of the second moment of the solution in large time for all compact Riemannian manifolds will be shown. The large time existence of higher moments on all compact manifolds is proven, confirming a previous result by Tindel and Viens with more restrictive initial conditions. In small time, the difficulty induced by nonuniqueness of geodesics will be presented, along with how to overcome it on manifolds of nonpositive curvature.

A Unified Approach to the Small-Time Behavior of the Spectral Heat Content for Isotropic Lévy Processes

Hyunchul Park (SUNY New Paltz)

Abstract: We establishes the precise small-time asymptotic behavior of the spectral heat content for isotropic Lvy processes on bounded C1,1 open sets of Rd with d ? 2, where the underlying characteristic exponents are regularly varying at infinity with index ? ? (1,2], including the case ? = 2. Moreover, this asymptotic behavior is shown to be stable under an integrable perturbation of its Lvy measure. These results cover a wide class of isotropic Lvy processes, including Brownian motions, stable processes, and jump diffusions, and the proofs provide a unified approach to the asymptotic behavior of the spectral heat content for all of these processes.

A Generalized Bernoulli Process and Fractional Binomial Distribution

Jeonghwa Lee (University of North Carolina Wilmington)

Abstract: Long-range dependence (LRD) refers to a phenomenon of a strong dependence in a sequence of random variables such that it has a different large-scale behavior than other stationary stochastic processes whose correlation decays exponentially fast. In this talk, I will introduce a generalized Bernoulli process (GBP) which consists of binary variables that can possess LRD. The fractional binomial distribution is defined from the sum of variables in GBP, and its applications in over-dispersed/excess zero count data will be examined. The extension of GBP to a finite state stationary process and fractional multinomial distribution will also be discussed.

Chung's Law of the Iterated Logarithm for a Class of Stochastic Heat Equations

Jiaming Chen (University of Rochester)

Abstract: We establish a Chung-type law of the iterated logarithm for the solution to a class of stochastic heat equations driven by a multiplicative noise whose coefficient depends on the solution, and this dependence takes us away from the Gaussian setting. Based on literatures on small ball probability and the freezing technique, the limiting constant can be evaluated almost surely.

Characterizations Of Variogram and Pseudo Variogram Matrix Functions

Juan Du (Kansas State University)

Abstract: Both covariance- and variance- based variogram matrix function are important measures for the dependence of a vector random field with second-order increments, and are useful tools for linear predication or cokriging. We will characterize both types of variogram matrix functions and show their inherent connection with covariance matrix functions. Various flexible dependence structures among different components are derived through those characterizations and appropriate mixture procedures. Particularly, we investigate the general forms of a power exponential model, a power-law pseudo variogram matrix function, and a vector fractional Brownian motion over a metric space.

The Dynamical Ising-Kac Model Converges to $\Phi 4$ in Three Dimensions

Konstantin Matetski (Michigan State University)

Abstract: The Glauber dynamics of the Ising-Kac model describes the evolution of spins on a lattice, with the flipping rate of each spin depending on an average field in a large neighborhood. Giacomin, Lebowitz, and Presutti conjectured in the 90s that the random fluctuations of the process near the critical temperature coincide with the solution of the dynamical $\Phi 4$ model. This conjecture was proved in one dimension by Bertini, Presutti, Ruediger, and Saada in 1993 and the two-dimensional case was proved by Mourrat and Weber in 2014. Our result settles the conjecture in the three-dimensional

case. The dynamical Φ4 model is given by a non-linear stochastic partial differential equation which is driven by an additive space-time white noise and which requires renormalization of the non-linearity in dimensions two and three. The renormalization has a physical meaning and corresponds to a small shift of the inverse temperature of the discrete system away from its critical value. This is joint work with Hendrik Weber and Paolo Grazieschi.

Matching Moment Upper and Lower Bounds for the One-Dimensional Stochastic Wave Equation

Le Chen (Auburn University)

Abstract: In this talk, we present a recent joint-work with Yuhui Guo and Jian Song (preprint available at arXiv:2206:10069). In this work, we study a class of nonlinear stochastic partial differential equation subject to multiplicative space-time white noise, with the stochastic heat equation (SHE) and the stochastic wave equation (SWE) as two special cases. The large-time exact p-th moment growth rates are known for SHE. However, the corresponding results for SWE have remained absent in the literature. In this study, we fill up this gap by demonstrating that for SWE,

$$t^{-1}\log\mathbb{E}\left(u(t,x)^p\right)\asymp p^{3/2},$$

as $t\to\infty$ for all $p\geq 2$. The main challenge lies in the lower bound estimate. Our method is based on some Feynman-Kac-type formula, inspired by the recent work of Hu and Wang in 2022, who studied the moment growth rates with noises that are both spatially and temporally colored. The space-time white noise case requires a separate treatment, which constitute one of the main contributions of the paper. The formulas we derived have opened up new avenues for further exploration and raised some intriguing questions for future research.

Dirichlet Fractional Gaussian Fields on the Sierpinski Gasket

Li Chen (Louisiana State University)

Abstract: We study Dirichlet fractional Gaussian fields on the Sierpinski gasket. Heuristically, such fields are defined as distributions $X_s=(-\Delta)^{-s}W$, where W is a Gaussian white noise and Δ is the Laplacian with Dirichlet boundary condition. The construction is based on heat kernel analysis and spectral expansion. We also discuss regularity properties and discrete graph approximations of those fields. This is joint work with Fabrice Baudoin.

Statistical Tractography

Lyudmila Sakhanenko (Michigan State University)

Abstract: I will discuss current progress in statistical tractography, which in the collection of integral curve estimation techniques, that are motivated by tractography problems from Diffusion Tensor Imaging and High Angular Resolution Diffusion Imaging in neuroscience. All the presented techniques come with theoretical guarantees from the asymptotical analysis of the corresponding stochastic processes.

Weighted l_1-penalized Corrected Quantile Regression for High-Dimensional Temporally Dependent Measurement Errors

Nilanjan Chakraborty (Washington University in Saint Louis)

Abstract: This paper derives some large sample properties of weighted l_1-penalized corrected quantile estimators of the regression parameter vector in a high-dimensional errors in variables (EIVs) linear regression model. In this model, the number of predictors p depends on the sample size n and tends to

infinity, generally at a faster rate than n, as n tends to infinity. Moreover, the measurement errors in the covariates are assumed to have linear stationary temporal dependence and known Laplace marginal distribution while the regression errors are assumed to be independent non-identically distributed random variables having possibly heavy tails. The paper discusses some rates of consistency of these estimators, a model consistency result and an appropriate data adaptive algorithm for obtaining a suitable choice of weights. A simulation study assesses the finite sample performance of some of the proposed estimators. This paper also contains analogs of Massart's inequality for independent and short memory moving average predictors, which is instrumental in establishing the said consistency rates of the above mentioned estimators in the current setup of high dimensional EIVs regression models.

This is a joint work with Prof. Monika Bhattacharjee and Prof. Hira Koul.

Large Deviations for Small Noise Diffusions Over Long Time

Pavlos Zoubouloglou (UNC Chapel Hill)

Abstract: We study two problems. First, we consider the large deviation behavior of empirical measures of certain diffusion processes as, simultaneously, the time horizon becomes large and noise becomes vanishingly small. The law of large numbers (LLN) of the empirical measure in this asymptotic regime is given by the unique equilibrium of the noiseless dynamics. Due to degeneracy of the noise in the limit, the methods of Donsker and Varadhan (1976) are not directly applicable and new ideas are needed. Second, we study a system of slow-fast diffusions where both the slow and the fast components have vanishing noise on their natural time scales. This time the LLN is governed by a degenerate averaging principle in which local equilibria of the noiseless system obtained from the fast dynamics describe the asymptotic evolution of the slow component. We establish a large deviation principle that describes probabilities of divergence from this behavior. On the one hand our methods require stronger assumptions than the nondegenerate settings, while on the other hand the rate functions take simple and explicit forms that have striking differences from their nondegenerate counterparts.

Fractional Brownian Motion: Small Increments and First Exit Time from One-sided Barrier

Qidi Peng (Claremont Graduate University)

Abstract: Based on an optimal rate random wavelet series representation, we derive a local modulus of continuity result with a refined almost sure upper bound for fractional Brownian motion. This result fills some gap in the law of iterated logarithm for fractional Brownian motion, by giving the moments' control of the almost sure upper bound of fractional Brownian increments. With this enhanced upper bound and some new results on the distribution of the maximum of fractional Brownian motion, we obtain a new and refined asymptotic estimate of the upper-tail probability for the fractional Brownian motion to first exit from a positive-valued barrier.

Optimal Gaussian Approximation for Stationary Spatial Fields

Soham Bonnerjee (University of Chicago)

Abstract: Gaussian approximation results are abound in the literature for i.i.d. random variables. Building on the ubiquitous Central Limit Theorem, extensive generalizations have been proposed, culminating in the seminal work by Komlos, Major, and Tusnady. Similar results have been obtained for stationary and even nonstationary random variables, however, corresponding results for random fields are yet to be obtained. We attempt to derive a strong approximation result for stationary random field with a causal structure. Such results have important applications. Building on our gaussian approximations, one can derive theoretical results for spatial change-point detection, a problem which despite

its importance has only scarce theoretical treatment. We further substantiate our theory with extensive simulations.

Random Flows on Spheres

Tianshi Lu (Wichita State University)

Abstract: We characterized the cross-covariance of an isotropic random flow on a hypersphere and obtained the Helmholtz-Hodge decomposition of the flow. We also derived the Karhunen-Loéve expansion of an isotropic Gaussian random flow on a hypersphere. On the 2-sphere, we characterized the rotationally invariant random flow, which allowed correlation between the curl free part and the divergence free part of the flow.

Local Excursion of Conditional Gaussian Fields

Victor Amaya Carvajal (Duke University)

Abstract: The problem of estimating the level sets of a conditional Gaussian process turns out to be complicated as the conditional process lacks stationary properties. We propose a new method in which the Adler & Taylor theory is used locally, the expected value of the Euler characteristic of the level sets, for the estimation of these local probabilities. Our method is computationally efficient and we experimentally test its use through simulations.

Multi-point Lyapunov Exponents of the Stochastic Heat Equation

Yier Lin (University of Chicago)

Abstract: We study the Stochastic Heat Equation with multiplicative space-time white noise. Extensive research has already been conducted on the one-point Lyapunov exponents of this equation. In this talk, I will present how we can compute the multi-point Lyapunov exponents by leveraging a combination of integrability and probability. Additionally, we solve a non-trivial optimization problem as a byproduct.