COLLOQUIUM

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Complexity Penalization in Low Rank Matrix Regression

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A405 Wells Hall
10:20 a.m. - 11:10 a.m.
Refreshments: 10:00 a.m.

Abstract

We will discuss a problem of estimation of a large $m \times m$ Hermitian matrix A based on i.i.d. measurements

$$Y_j = tr(AX_j) + \xi_j, j = 1, ..., n,$$

where X_j are random $m \times m$ Hermitian matrices and $\{\xi_j\}$ is a zero mean random noise. The goal is to estimate A in the case when it has relatively small rank (or it can be well approximated by small rank matrices) with an error proportional to the rank. This problem has been extensively studied in the recent years, especially, in the case of noiseless measurements (e.g., Candes and Recht (2009); Candes and Tao (2009)). Important examples include matrix completion, when a random sample of entries of A is observed, and quantum state tomography, when A is a density matrix of a quantum system and the goal is to estimate A based on the measurements of n observables $X_1, ..., X_n$. We will consider several approaches to such problems based on a penalized least squares method (and its modifications) with complexity penalties defined in terms of nuclear norm, von Neumann entropy and other functionals that "promote" small rank solutions. We will discuss oracle inequalities for the resulting estimators with explicit dependence of the error terms on the rank and other parameters of the problem. Their proofs are based on a variety of tools including concentration inequalities, generic chaining bounds and noncommutative extensions of classical exponential bounds for sums of independent random variables.